Design-Oriented Parametrization of Truncated Periodic-Strip Gratings

Lawrence Carin, *Member, IEEE*, and Leopold B. Felsen, *Life Fellow, IEEE*

Abstract—**Spectral domain asymptotics are used to develop a hybrid (ray)-(Floquet mode) parametrization that models time-harmonic plane-wave interaction with a truncated grating of periodically spaced, coplanar, infinitesimally thin, perfectly conducting strips in free space. By distinctly displaying the edge effects as well as the truncated Floquent mode contributions from the body of the grating, the model—which is valid in the near and far zones—contains the necessary ingredients for finite-grating design; the truncated Floquent modes are based on those for the finite grating. Plane-wave diffraction results computed from the model are shown to agree very well with numerical reference data generated by a (spectral domain)-(method of moments) algorithm.**

RECENTLY, we have developed a hybrid (ray)-(Floquent mode)-(MOM) algorithm for numerical as well as analytic-asymptotic modeling of two-dimensional time-harmonic and transient plane-wave scattering from finite gratings composed of coplanar, infinitesimally thin, perfectly conducting strips in free space [1]; the method of moments (MOM) is used to determine the currents induced on the strips. The resulting explicit asymptotic fields have been found to agree remarkably well with direct numerical reference data. They parametrize the data in terms of dominant physical scattering mechanisms, which are uniformly valid for observations in the near, intermediate, or far zone of the grating. Involving edge diffractions from the ends of the grating, truncated Floquet modes from the bulk, and transition functions across the Floquet-mode shadow boundaries, these constituents furnish the wave-optical tools for finite-grating design. In this communication we demonstrate the method for time-harmonic scattering.

Consider the finite-periodic N-strip grating shown in Fig. 1, with strip width w, strip spacing s, and period $d = w + s$. We analyze time-harmonic plane-wave scattering from such a structure by first performing a MOM analysis for the induced currents [2]. After determining the currents, we evaluate the scattered fields by using the spectral domain versions of the Green's function and surface currents (Fourier transform with respect to the spatial variable x):

$$u_s(x,z) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \tilde{u}_G(k_x,z)\tilde{J}(k_x)e^{ik_x x}dk_{x'}, \qquad (1)$$

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The authors are with Weber Research Institute, Polytechnic University, Route 110, Farmingdale, NY 11735.

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where k_x is the spatial spectral variable, $\tilde{u}_G$ represents the spectral domain Green's function for scattered field component u_s, $\tilde{J}$ represents the surface currents in the spectral domain, and the implied e^{-iwt} time dependence is suppressed. The free-space spectral Green's function contains no poles in the complex k_x-plane.

The surface currents on all strips are found rigorously by the MOM, but are approximated as

$$J_n[x' - (n-1)d] \cong J_{n_0}[x' - (n-1)d]e^{ik_0 d(n-n_0)\sin\theta_i}, \qquad (2)$$

where $J_n[x' - (n-1)d]$ is the current on the nth strip centered at $x_n = (n-1)d$ and extending over $|x' - x_n| < \frac{w}{2}$, θ_i is the plane-wave incidence angle, $k_o = 2\pi/\lambda_o$ is the free-space wavenumber, and J_{n_o} is the MOM-computed current on the reference strip n_o that will be chosen to lie in the center of the grating. This approximation neglects end effects associated with the outer strips of the grating, but such effects have been found to be negligible in cases investigated so far.

By expressing the currents as in (2), it can easily be shown that $\tilde{J}$ has an infinite number of poles at $k_{xm} = k_0\left[\frac{m\lambda_0}{d} - \sin\theta_i\right]$, for all integers m. These poles correspond exactly to the discrete Floquet mode spectra of a corresponding *infinitely* wide grating. Using (2) in (1), we therefore introduce Floquet-mode poles into the inverse spectral integral. The scattered field can be expressed as

$$u_s = u_l - u_r \exp\left[-ik_o N d \sin\theta_i\right] \qquad (3)$$

with $u_{l,r}$ evaluated by uniform asymptotics [3] to yield an expression of the form

$$\begin{aligned} u_{l,r} \sim & \sqrt{2\pi/k_o L_{l,r}}e^{ik_o L_{l,r}}e^{-i\pi/4}g(\theta_{l,r},\theta_i) \\ & - \sum_m \operatorname{sgn}(\theta_{l,r} - \phi_m) i\pi a_m e^{ik_0 L_{l,r}|\phi_m - \theta_{l,r}|^2/2} \\ & \quad \operatorname{erfc}\left[\sqrt{k_o L_{l,r}/2}e^{-i\frac{\pi}{4}}|\phi_m - \theta_{l,r}|\right] \\ & + 2\pi i \sum_m a_m e^{ik_0 L_{l,r}\cos[\phi_m - \theta_{l,r}]} \\ & \quad U[|\theta_{l,r}| - |\phi_m|]U[\theta_{l,r}\phi_m]\operatorname{sgn}(\theta_{l,r}). \end{aligned} \qquad (4)$$

Here, erfc$(\cdot)$ denotes the error function complement, $U(\cdot)$ the Heaviside function, a_m the residue at the pole k_{xm}, and $\phi_m = \sin^{-1}\left[\frac{m\lambda_0}{d} - \sin\theta_i\right]$ the angle of propagation of the mth Floquet mode. Furthermore, $L_{l,r}$ and $\theta_{l,r}$ locate the observation point with respect to the outer edges of the left and right most unit cells of the truncated grating (see Fig. 1). In (4),

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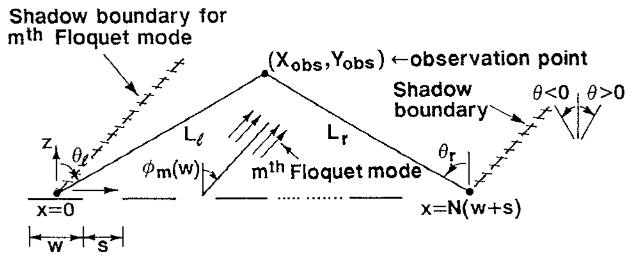

Fig. 1. Truncated periodic grating of N coplanar infinitesimally thin, perfectly conducting strips of width w and spacing s in free space. Coordinates and observation regions pertain to the GTD interpretation of plane-wave scattering from the grating. The angles $\theta_{l,r}$ and distances $L_{l,r}$ locate the left-most and right-most unit cells, respectively, as seen by the observer at $(x_{\text{obs}}, y_{\text{obs}})$. The angle $\phi_m(w)$ denotes the angle of propagation of the mth Floquet mode, its domain of existence being limited by the dashed shadow boundaries.

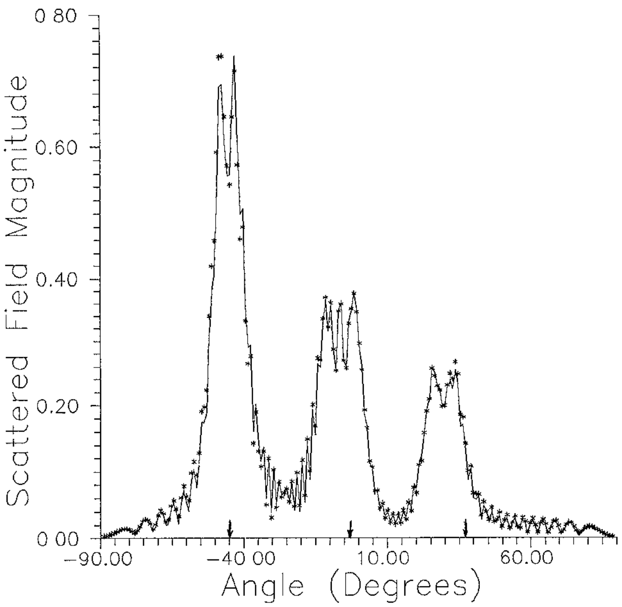

Fig. 2. Scattered electric field magnitude from 20 strips of width 1 λ_0 and spacing $\lambda_0/2$ due to a TM polarized plane wave incident at $\theta_i = 45°$, observed as a function of angle at distance 100 λ_0 from the grating center. Solid curve represents reference results from a direct numerical inversion; the points represent asymptotic results from (4). Arrows identify the angles $\phi_0 = -45°$, $\phi_1 = -2.31°$, $\phi_2 = 38.8°$ corresponding to the 3 Floquet modes ϕ_m excited on an infinite grating period $(w + s) = 1.5\ \lambda_o$.

we assume observation distances at which evanescent modes are negligible; this restricts the mode sum to $|k_{xm}| < k_0$.

The physical interpretation of (4) has been incorporated into Fig. 1. The third term in (4) represents Floquet modes which individually contribute in a semi-infinite spatial domain to the right of the respective shadow boundaries $\theta_{l,r} = \phi_m$. In the total scattered field (3), this implies the absence of the mth Floquet mode in the domain $\theta_l < \phi_m$ and $\theta_r > \phi_m$; in these regions, the scattering is due to contributions from edge diffraction and possibly other modes. The first and second terms in (4) are associated with wave phenomena that are phase centered at the outermost left-hand (subscript 1) and right-hand (subscript r) unit cells. Together, they describe edge effects and establish a uniform transition across the shadow boundary of each mode. Although the first expression in (4) has the form of a diffraction term in the geometrical theory of diffraction (GTD), the incorporation of uniform asymptotics removes its singularities at the shadow boundaries; therefore, it does *not* by itself represent classical GTD edge diffraction [4]. When the grating is allowed to become wider and the fields are observed not too far from the grating surface, the first two terms in (4) become negligible, and the fields are expressed by the remaining Floquet mode expansion characteristic of infinite gratings.

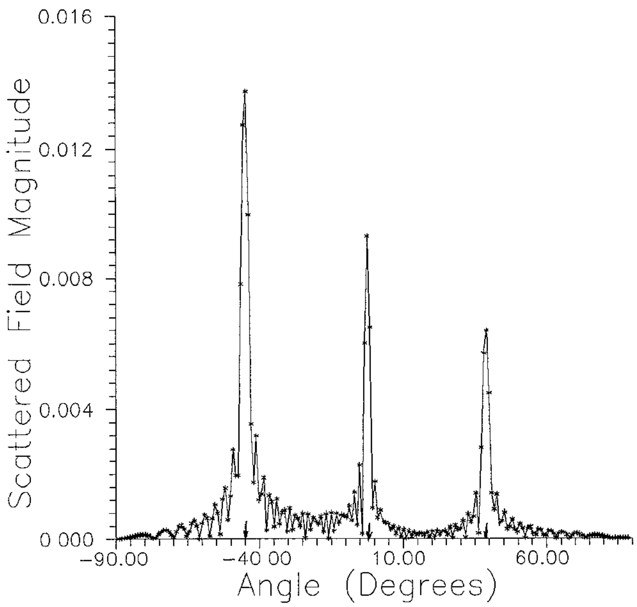

Fig. 3. As in Fig. 2, but with the fields observed at $10^6 \lambda_0$ from the grating center. Solid curve represents results from the numerical reference solution; the points are based on the simplified far-field-pattern function $f(\theta, \theta_i)$.

When the observer moves to the far zone of the grating where $\theta_l \approx \theta_r = \theta$, the individual shadow boundary transition regions overlap, and the expression in (4) changes in such a manner that the *total* scattered field in (3) can be characterized even at $\theta = \phi_m$ by a far-field-pattern function $f(\theta, \theta_i)$ synthesized *entirely* by the *nonuniform* (classical) edge diffraction from the left and right truncations. Each edge diffraction has the form of the first term in (4) provided that the partial edge-diffraction coefficients $g(\theta_{l,r}, \theta_i)$ are replaced by the full versions $h(\theta_{l,r}, \theta_i)$, $\theta_{l,r} \to \theta$. For TM polarization, one finds from far-field asymptotics applied to (1) that

$$h(\theta_{l,r}, \theta_i) = \frac{\tilde{J}_1(k_0 \sin\theta_{l,r}) \cos\theta_{l,r}}{[1 - \exp(-idk_0(\sin\theta_i + \sin\theta_{l,r}))]}. \quad (5)$$

Although each $h_{l,r}$ is singular at $\theta_{l,r} = \phi_m$, the singularities cancel when combined to construct u_s (see [4] for the classical case). Analogous comments apply to TE polarization. Since the diffraction coefficients depend on the spectral amplitudes $\tilde{J}_1(k_0 \sin\theta_{l,r})$ of the strip currents as well as on the dimension d of the unit cell, they carry the signature of the *infinite* grating.

To exhibit the accuracy of (4) and of its reduced far-field version, we consider scattering from a 20-strip grating with $w = \lambda_0$ and $s = \frac{1}{2}\lambda_0$, excited by a TM polarized plane wave incident at an angle of 45°. The electric field magnitude observed 100 λ_0 from the center of the grating is plotted as a function of angle in Fig. 2. The solid curve was obtained by numerical integration of (1) with the MOM-computed currents from all 20 strips, while the dots represent results computed from (4). In this near zone of the grating, θ_l and θ_r are distinct, therefore requiring the detailed uniform asymptotics in (4). Fig. 3 contains results for the same problem when the observation circle is in the far zone observed $10^6\lambda_0$ from the center of the grating. Again, the numerical reference solution shown solid in Fig. 3 was computed from (1) with the individual currents on all 20 strips. The dots are from the simplified far-field form that involves $f(\theta, \theta_i)$. Clearly evident in both figures are peaks near the propagation angles $\phi_0 = -45°$, $\phi_1 = -2.31°$, and $\phi_2 = 38.8°$ of the three Floquet modes that are excited on an infinite grating with the same strip width and spacing. Clearly evident also is the loss of fine resolution in scattered field data (Fig. 2) when the observer moves to the far zone (Fig. 3).

The results here, which are typical samples from a large data bank, demonstrate the utility of the hybrid (ray)–(Floquet mode)–(MOM) algorithm for calculating and physically interpreting the fields scattered from a finite periodic strip grating. The synthesizing wave objects describe edge diffraction from the ends of the grating, bulk scattering in terms of finite-aperture Floquet modes, and transition functions across the Floquet-mode shadow boundaries. The signature of the periodicity is imprinted throughout and is particularly "clean" in the far-zone diffraction pattern. Representing generalizations, for periodic structure scatterers, of their conventional GTD counterparts for smooth truncated surfaces, and being remarkably accurate as demonstrated by comparison with numerical reference data, the new wave objects should find use in the analysis and design of finite gratings just as conventional GTD wave constituents find use in the analysis and control of target scattering.

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